Hamilton-Jacobi-Bellman equation for Rydberg-blockade processes

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Optically trapped neutral atoms are one of the most promising platform for quantum computation due to their controllability and scalability. By means of optical tweezers, atoms can be arranged into arrays, and qubit degrees of freedom can be encoded in the electronic states of the atoms. Then, multiqubit operations can be realized exploiting the strong van der Waals interactions among high principal quantum number states, via the so-called Rydberg blockade mechanism. Currently, spontaneous emission from excited states represents the predominant source of infidelity, generating incoherent errors whose magnitude increases with the process duration. This makes the design of *time-optimal* protocols of key importance, and recent efforts have focused on optimizing pulse shapes for entangling gates, either through gradient descent methods based on time discretization or through analytical ansatz-based optimization [1, 2].

In this work, we present the Hamilton-Jacobi-Bellman (HJB) formalism as a natural theory for seeking global optima in time-optimal control problems with globally-driven Rydberg arrays. Given a target set of states, the optimal cost function, namely the minimum time needed to reach the target from any state, satisfies the so-called HJB equation [3]. This equation is derived from the Bellman principle of optimality [4] and constitutes a necessary and sufficient condition for the global optimum. It is a nonlinear first-order partial differential equation (PDE) that admits a unique generalized viscosity solution [5], from which optimal trajectories and controls can be directly generated. Not making any a priori assumption on the control's shape, this method naturally returns the global optimum of the problem. We validate the Bellman approach for time-optimal control problems in globally-driven Rydberg platforms as follows. On one side, we exploit this method to optimize several processes within the standard setting of Rydberg entangling gates, among which we reproduce the known results for controlled-phase gates. On the other side, we focus on the universal quantum computation scheme introduced in [6], and compute optimal pulses for its fundamental building blocks.

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