# A scalable quantum switch for a genuine indefinite causal order

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The quantum switch is a higher-order quantum process which takes as input $N$ quantum gates and a quantum state, a applies the N gates to the input state in a superposition of all possible $N!$ permutations [1]. The quantum switch is interesting both for the fundamental interest in creating processes with a so-called indefinite causal order [2], and for its ability to provide advantages beyond standard quantum circuits [3]. Creating the N-switch for $N > 2$ is important both for applications and fundamental interest. From the fundamental point-of-view, this would allow for the study of multipartite indefinite causality, which has a similar mathematical structure to multipartite entanglement. While from the applications side, many applications will require the creation of the N-switch to provide a truly useful advantage.

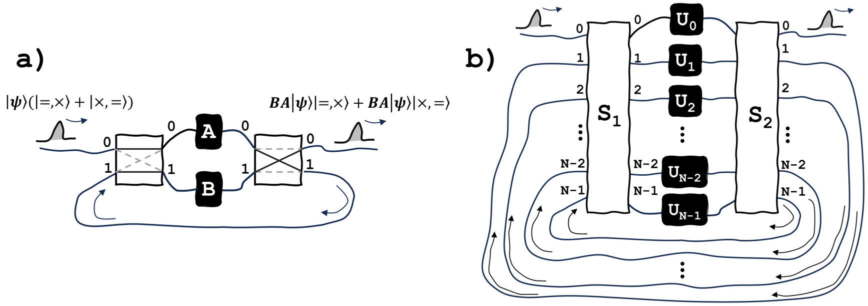

**Fig. 1:** a) The two-switch made up of two quantum routers. When a router is in the 'bar state ' $|{=}\rangle$ the modes are transmitted, while the 'cross state' $|\times\rangle$ results in the modes being swapped. Thus, if a photon is incident in a single spatio-temporal mode and the two routers begin in the entangled state $(|{=},\times\rangle + |\times,{=}\rangle)/\sqrt{2}$ the operations $A$ and $B$ are applied in a superposition of both orders. In other words, the quantum switch is realized. b) Scaling up the idea to N gates. Now two NxN quantum routers are used. When they begin in a suitable entangled state the N-switch is realized.

In the N-switch the specific permutation to be applied is determined by a control register, meaning that in order to implement the N-switch at least an $N!$-dimensional control system is required. Current experimental implementations of the quantum switch use a single photon to encode both the control and target systems. Typically the target system is taken to be some internal degree of freedom, such as the polarization state of the photon, and the control system is encoded in the path of the same

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photon [4]. In practice, this means that the photon must be placed in a superposition of $N!$ different optical paths, and that each path must then be routed through the optical is a given order. Given this combinatorial explosion in the number of required paths, experimental implementations of the full the quantum switch have thus far been limited to $N = 2$.

Here we show that if the control system is instead encoded in a different physical system, one can physically realize the quantum switch in a resource-efficient way. In particular, as illustrated in the figure, we imagine a system of quantum routers which are used to coherently rout a photon into different paths. However, unlike a classical beamsplitter, the state of the quantum router becomes entangled with path of the photon. Such quantum routers have already been realized experimentally, using single-photon level nonlinearities [5]. In Fig. 1a we show how two such quantum routers can be used to implement the quantum switch for N=2. Fig. 1b shows how this can be achieved for N gates, using instead two NxN quantum routers. Each NxN quantum routers can be realized using $N^2$ 2x2 quantum routers. Thus the number of resources required for our proposal scales as $N^2$, rather than as $N!$ as required in previous proposals. Furthermore, for many simplifications of our scheme are possible, and we will present these in our presentation. Although these are certainly challenging experiments, it is also possible to realize the 2 switch with a single quantum router, making it fairly straightforward to implement this with current capabilities.

Finally, we note that in addition to the scaling advantages, our proposal has a number of other foundational advantages. For one, in our proposal the control system does not need to traverse the gates along with the target photon. This has been criticized as a loophole to realizing a genuine indefinite causal order in current experimental implementations [6]. Furthermore, our proposal only requires one mode to traverse each gate, which is another past experimental work [7]. Thus from a foundational point of view, our proposal also shows that it is possible to realize a genuine indefinite causal order in a quantum optical setting, and more ‘exotic physics,’ is not required as suggested by some [8].

## References

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