Towards quantum advantage via Topological Data Analysis

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Even after decades of quantum computing development, examples of generally useful quantum algorithms with exponential speedups over classical counterparts are scarce. Recent progress in quantum algorithms for linear-algebra positioned quantum machine learning (QML) as a potential source of such useful exponential improvements. Yet, in an unexpected development, a recent series of "dequantization" results has equally rapidly removed the promise of exponential speedups for several QML algorithms [1]. This raises the critical question whether exponential speedups of other linear-algebraic QML algorithms persist. In this paper, we study the quantum-algorithmic methods behind the algorithm for topological data analysis [2] through this lens. We provide evidence that the problem solved by this algorithm is classically intractable by showing that its natural generalization is as hard as simulating the one clean qubit model [3] – which is widely believed to require superpolynomial time on a classical computer – and is thus very likely immune to dequantizations. To analyze whether it is possible to further strengthen the argument for quantum advantage (or, to actually find an efficient classical algorithm), we analyze the theoretical hurdles that, at least currently, stymie state-of-the-art classical algorithms from performing equally well as the quantum algorithm. Based on the above result, we provide a number of new quantum algorithms for problems such as rank estimation and complex network analysis, along with complexity-theoretic evidence for their classical intractability. Furthermore, we analyze the suitability of the proposed quantum algorithms for near-term implementations. In particular, we discuss techniques aimed at reducing the required number of qubits and we estimate that using our techniques a total of approximately 80 qubits are potentially enough to challenge the best known classical methods. Our results provide a number of useful applications for full-blown, and restricted quantum computers with a guaranteed exponential speedup over classical methods, recovering some of the potential for linear-algebraic QML to become one of quantum computing's killer applications.

References

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