## **Resource Theory of Causal Connection**

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The capacity of distant parties to send one another signals is a fundamental requirement in many information-processing tasks. Such ability is determined by the causal structure connecting the parties, and more generally, by the intermediate processes carrying signals from one laboratory to another. Here we build a fully fledged resource theory of causal connection for all multi-party communication scenarios, encompassing those where the parties operate in a definite causal order, and also where the order is indefinite. We define and characterize the set of free processes and three different sets of free transformations thereof, resulting in three distinct resource theories of causal connection. In the causally ordered setting, we identify the most resourceful processes for two- and three-party scenarios. In contrast, our results suggest that in the general setting there is no global most valuable resource. Finally, we provide a monotone of causal connection, the signalling robustness, and derive tight bounds thereof. Our results offer a flexible and comprehensive framework to quantify and transform general quantum processes, as well as insights into their multi-layered causal connection structure.



Fig. 1: General transformations  $\Upsilon$  of causally (dis-)ordered quantum processes W.

## References

[1] S. Milz, J. Bavaresco, and G. Chiribella, arXiv:2110.03233 (2021).

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